Analytical and numerical results on the attenuation and dispersion of an acoustic wave through a two-phase flow

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Nomenclature

Each variable $\bar{X}$ is written as the sum of a mean X_0 value and a fluctuation term X.

A_i = maxima of the signal
c = speed of sound in two-phase medium
c_0 = speed of sound in the gas
C_d = drag coefficient
C_m = mass concentration
C_p = specific heat of gas at constant pressure
C_v = specific heat of gas at constant volume
D_p = particle diameter
f_0 = frequency
n = particles number per unit volume
P = gas pressure
Pr = Prandtl number
Q_p = heat transferred to one particle
R = gas constant
Re = Reynolds number
t = time
T = temperature
Ts = temporal period
F = flux termu
F_p = drag force
H = source term
K = complex wave propagation number
k_1, k_2 = real and imaginary parts of the number K
m' = mass of one particle
u = axial velocity
U = conservative term
ω = circular acoustic frequency
x = duct abscissa
Z = velocities ratio

Greek symbols

α_p = volume concentration of particles
α' = coefficient of energy attenuation
$\tilde{\alpha}$ = dimensionless attenuation
β = dispersion coefficient
$\tilde{\beta}$ = dimensionless dispersion
$\Delta 1$ = distance between two maxim
ΔP = amplitude of pressure
ε = total energy
γ = ratio of specific heats of the gas
λ = thermal conductivity of gas
μ = gas viscosity
ρ = density
τ_d = dynamic relaxation time of particle
τ_t = thermal relaxation time of particle
Φ = phase lag of the particles and gas velocities ratio

Subscripts

diph = two-phase flow
0 = mean value
p = particles

1. Introduction

Gas-particles interactions in two-phase mixtures have been widely investigated during the last decade both from analytical and numerical points of view. This

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work comes within the frame of the study of the disturbances caused by particles in an acoustic field[1,2].

Analytical solutions based on the linearization of the basic governing equations of two-phase flows have been proposed by Temkin and Dobbins[3] in a quite general scope. Such an approach makes it possible to obtain the actual values of sound velocity, the modulus and the phase lag of the velocities ratio u_p/u.

Quite a similar approach was developed by Culick[4] in studies concerning the instability phenomena occurring in rocket engines. In such flows excitation phenomena can occur, like those produced by the "combustion-flow" coupling effects or associated with the geometry of the combustion chamber[5]. The limited performance of these theoretical studies required numerical modelling. Several classes of methods have been used: finite volumes schemes (Roe), and finite differences methods (Mac-Cormack).

The aim of this study is to verify, in a simple configuration (1D), whether centred schemes are able to compute acoustic phenomena correctly in two-phase flows or not, in order to extend computations to multidimensional flows. Basic equations are linearized around a mean velocity u0. Numerical and analytical results are compared. The numerical scheme retained is the Mac-Cormack one in its finite volume formulation.

2. Mathematical model

2.1 Basic equations

In order to simulate unsteady two-phase dilute flows the "two fluids" model is generally used. Each phase is treated as a continuum medium; dynamics is described by an Eulerian approach. The basic equations governing such a problem are deduced from balance equations (mass, momentum, energy).

The main assumptions are:

- The gas phase is compressible and obeys the ideal gas law.
- The gas is inviscid (except the gas-particle interface).
- Particles are spherical and have a uniform temperature.
- The dispersed phase is very dilute: the volume forces of this phase can be neglected[6,7].
- As a consequence, the set of governing equations of the dispersed phase is weakly coupled with the gas one by source terms only.
- Interactions between gas and particles occur only by the way of drag forces and heat transfers.
- Any break up, coalescence of droplets and interactions (e.g. droplet-droplet, droplet-wall) are ignored.

$\bar{\rho}_p$ is the particle apparent density. The global set of equations in a conservative form can be written as follows:

Gas phase:

$$\frac{\partial}{\partial t}\begin{bmatrix}\bar{\rho}\\ \bar{\rho}\,u\\ \bar{\rho}\,\varepsilon\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\bar{\rho}\,\bar{u}\\ \bar{\rho}\,\bar{u}^2+\bar{P}\\ (\rho\,\varepsilon+\bar{P})\bar{u}\end{bmatrix}=\begin{bmatrix}0\\ n\,F_p\\ \bar{n}\,(Q_p+F_p\,\bar{u}_p)\end{bmatrix} \tag{1}$$

Dispersed phase:

$$\frac{\partial}{\partial t}\begin{bmatrix}\bar{\rho}_p\\ \bar{\rho}_p\,\bar{u}_p\\ \bar{\rho}_p\,\varepsilon_p\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\bar{\rho}_p\,\bar{u}_p\\ \bar{\rho}_p\,\bar{u}_p^2\\ (\bar{\rho}_p\,\varepsilon_p)\,\bar{u}_p\end{bmatrix}=\begin{bmatrix}0\\ -\bar{n}\,F_p\\ -\bar{n}\,(Q_p+F_p\,\bar{u}_p)\end{bmatrix}$$

where F_p is the drag force, and Q_p is the heat transfer. These terms are evaluated from usual correlations.

2.2 Transfer terms expression

For one particle, the drag force in the numerical simulation is taken as:

$$F_p=\frac{1}{2}\,C_d\,\pi\,R^2\,\rho\,|u-u_p|\,(u_p-u) \tag{2}$$

and the drag coefficient is issued from the relation:

$$C_d=\mathrm{Max}\left(\frac{24}{Re}\,;\,\frac{18{,}5}{Re^{0{,}6}}\,;\,0{,}44\right) \tag{3}$$

where:

$$Re=\frac{\rho\,|u-u_p|\,D}{\mu} \tag{4}$$

is the particulate Reynolds number.

The heat transfer is given by:

$$Q_p=\frac{\rho_p\,C_{pp}}{n_0}\,\frac{T_p-T}{\tau_t} \tag{5}$$

where τ_t is the thermal relaxation time.

The Stokes relation gives the expression of the drag force for one spherical particle at constant velocity for low Reynolds numbers. As soon as a particle is subjected to a perturbation, other forces resulting from the fluid motion must be added to the Stokes term. These forces are the pressure gradient force, the apparent mass force and the Basset force. As the densities ratio of gas and particles is less than unity, all these forces can be neglected in comparison with the Stokes term[8]. Furthermore, the forces acting crosswise on a particle, such

as the Magnus force or the shearing force, can also be neglected in front of the Stokes force as long as the diameter of particle remains small.

In the present case equations (2) and (5) become respectively:

$$F_p = 6 \pi \mu R (u_p - u)$$

$$Q_p = 4 \pi R \lambda (T_p - T)$$

These expressions are valid for Stokes velocity.

3. Theoretical method

3.1 Linear equations: general case

Analysis is based on the linearization of the set of one dimensional equations issued from the moving media acoustic theory. The main assumptions retained for this study are the following:

- Pressure, density, temperature and velocity disturbances due to the acoustic wave are small compared to their mean values, and the square or cross product of the disturbance are neglected in front of the first order terms.
- The work of the friction forces is neglected in the energy balance equation.

Consider a one dimensional duct, in which an acoustic wave propagates through a two-phase flow. Gas and particles dynamic equations (1) turn into the following form:

Gas phase:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\,\bar{u}}{\partial x} = 0$$

$$\bar{\rho}\left[\frac{\partial \bar{u}}{\partial t} + \bar{u}\,\frac{\partial \bar{u}}{\partial x}\right] + \frac{\partial \bar{P}}{\partial x} = \bar{n}\,F_p$$

$$\bar{\rho}\,C_v\left[\frac{\partial \bar{T}}{\partial t} + \bar{u}\,\frac{\partial \bar{T}}{\partial x}\right] + \bar{P}\,\frac{\partial \bar{u}}{\partial x} = \bar{n}\,Q_p \qquad (6)$$

Dispersed phase:

$$\frac{\partial \overline{\rho_p}}{\partial t} + \frac{\partial \overline{\rho_p}\,\overline{u_p}}{\partial x} = 0$$

$$\overline{\rho_p}\left[\frac{\partial \overline{u_p}}{\partial t} + \overline{u_p}\,\frac{\partial \overline{u_p}}{\partial x}\right] = -\,\bar{n}\,F_p$$

$$\overline{\rho_p}\,C_v\left[\frac{\partial \overline{T_p}}{\partial t} + \overline{u_p}\,\frac{\partial \overline{T_p}}{\partial x}\right] = -\,\bar{n}\,Q_p \qquad (7)$$

Starting from an initial state, where both phases are in equilibrium, supposing that flow is average and temperature is uniform, linearized equations for small disturbances of the variable $\bar{X}$ are found. Each variable $\bar{X}$ is written as the sum of a mean value X_0 and a fluctuation term X. One can deduce, for the gas phase:

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} + u_0 \frac{\partial \rho}{\partial x} = 0$$
$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} + \rho_0 u_0 \frac{\partial u}{\partial x} = n_0 F_p$$
$$\rho_0 C_v \frac{\partial T}{\partial t} + P_0 \frac{\partial u}{\partial x} + \rho_0 u_0 C_v \frac{\partial T}{\partial x} = n_0 Q_p \tag{8}$$

and for the dispersed phase, with $m' = \rho_{p0}/n_0$

$$\frac{\partial \rho_p}{\partial t} + \rho_{p0} \frac{\partial u_p}{\partial x} + u_{p0} \frac{\partial \rho_p}{\partial x} = 0$$
$$\rho_{p0} \frac{\partial u_p}{\partial t} + \rho_{p0} u_{p0} \frac{\partial u_p}{\partial x} = - n_0 F_p$$
$$\rho_{p0} C_{pp} \frac{\partial T_p}{\partial t} + \rho_{p0} u_{p0} C_{pp} \frac{\partial T_p}{\partial x} = - n_0 Q_p \tag{9}$$

The ideal gas law can be expressed as:

$$P = \acute{R} (\rho + \rho_0)(T + T_0) = \acute{R} (\rho_0 T_0 + \rho_0 T + \rho T_0) \tag{10}$$

Assuming that T, ρ, u and ρ_p are functions of the term $e^{i(Kx-\omega t)}$, the linearized problem can be solved.

The momentum equation for particles leads to:

$$m' \frac{\partial u_p}{\partial t} = - F_p - u_0 m' \frac{\partial u_p}{\partial x}$$

Using results of the velocity derivatives and introducing the drag force expression, the particle velocity is then given by the relation:

$$u_p = - \frac{u_p - u}{\dfrac{m'}{6 \pi \mu R} [- i (\omega - K u_0)]} = - \frac{u_p - u}{- i \tau_d (\omega - K u_0)} \tag{11}$$

with

$$\tau_d = \frac{m'}{6 \pi \mu R} = \frac{2 \rho_p R^2}{9 \mu} \tag{12}$$

Using the heat transfer between phases relation, one can find the particle energy balance equation in such a form:

$$T_p = - \frac{T_p - T}{\frac{m' C_{pp}}{4 \pi \lambda R}[- i (\omega - K u_0)]} = \frac{T_p - T}{- i \tau_t (\omega - K u_0)} \tag{13}$$

with

$$\tau_t = \frac{m' C_{pp}}{4 \pi \lambda R} = \frac{2}{3} Pr \frac{C_{pp}}{C_p} \tau_d \tag{14}$$

From equations (11) and (13) it follows that:

$$u_p = \frac{u}{1 - i \tau_d (\omega - K u_0)} \tag{15}$$

$$T_p = \frac{T}{1 - i \tau_t (\omega - K u_0)} \tag{16}$$

Thus the set of equations can be written as:

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} + u_0 \frac{\partial \rho}{\partial x} = 0 \tag{17}$$

By elimination of the pressure in the gas momentum equation and using the drag force expression, the momentum equation is obtained in the next form:

$$\frac{\partial u}{\partial t} + \frac{\hat{R} T_0}{\rho_0} \frac{\partial \rho}{\partial x} + \hat{R} \frac{\partial T}{\partial x} + u_0 \frac{\partial u}{\partial x} = \frac{i u C_m (\omega - K u_0)}{1 - i \tau_d (\omega - K u_0)} \tag{18}$$

with

$$C_m = \frac{n_0 m'}{\rho_0} = \frac{n_0}{\rho_0} 6 \pi \mu R \tau_d$$

In the same way, elimination of the pressure in the gas energy equation leads to:

$$\rho_0 C_v \frac{\partial T}{\partial t} + P_0 \frac{\partial u}{\partial x} + \rho_0 u_0 C_v \frac{\partial T}{\partial x} = n_0 Q_p$$

and, substituting the expression for heat transfer expression one can derive:

$$\frac{\partial T}{\partial t} + \frac{P_0}{\rho_0 C_v} \frac{\partial u}{\partial x} + u_0 \frac{\partial T}{\partial x} = \gamma \frac{C_{pp}}{C_p} C_m \frac{i (\omega - K u_0) T}{1 - i \tau_t (\omega - K u_0)} \tag{19}$$

Then, the particle mass equation, introducing the relation (15), becomes:

$$\frac{\partial \rho_p}{\partial t} + \frac{C_m \rho_0}{1 - i \tau_d (\omega - K u_0)} \frac{\partial u}{\partial x} + u_{p0} \frac{\partial \rho_p}{\partial x} = 0 \tag{20}$$

This new set of equations (17-20) may have a solution for (T, ρ, u, ρ_p) if the following determinant is equal to zero:

$$\begin{bmatrix} 0 & \Omega & K\rho_0 & 0 \\ K\tilde{R} & \frac{K\tilde{R}T_0}{\rho_0} & \Omega\left(1+\frac{C_m}{1-i\Omega\tau_d}\right) & 0 \\ \Omega\left(1+\gamma\frac{C_{pp}}{C_p}\frac{C_m}{1-i\Omega\tau_t}\right) & 0 & K(\gamma-1)T_0 & 0 \\ 0 & 0 & \frac{K C_m \rho_0}{1-i\Omega\tau_d} & \Omega \end{bmatrix}$$

with: $\Omega = -\omega + K u_0$ and $K = k_1 + i k_2$

At this moment, the resolution of such a determinant is done, in the general case, by a numerical procedure. Indeed, owing to the complex number "– w + K u_0", involving the initial velocity, development of the determinant yields a fourth degree complex polynomial. The solution of such an equation in order to obtain analytical expression of the real and imaginary part[3] is not obvious and requires a numerical treatment and some constraints. The selection criteria required for the solution are:

- both real (k_1) and imaginary (k_2) parts of the complex wave number must be positive (see relations 26 and 27).
- the modulus of the complex wave number must be close to:

$$|K| \approx \frac{\omega}{c_0 + u_0} \tag{21}$$

It is supposed that these criteria are sufficient to choose the appropriate solution.

3.2 Particular case: $u_0 = 0$

This case is interesting because of its analytical solution previously found by Temkin and Dobbins[3]. The solution is obtained by putting $u_0 = {}_0$ (or $\Omega = -\omega$) into the previous determinant. The calculation of the determinant in that case leads to the following relation:

$$\left(\frac{K c_0}{\omega}\right)^2 \left(1 + \frac{C_{pp} C_m}{C_p (1 - i\omega\tau_t)}\right) = \left(1 + \frac{C_m}{(1 - i\omega\tau_d)}\right)\left(1 + \frac{\gamma C_{pp} C_m}{C_p (1 - i\omega\tau_t)}\right) \tag{22}$$

In order to compute k_1 and k_2, one sets:

$$f(\omega) = k_1^2 - k_2^2 \tag{23}$$

$$g(\omega) = k_1 k_2 \tag{24}$$

It becomes easy to deduce f(ω) and g(ω) from the real and imaginary parts of equation (22).

Then, solving a biquadratic equation one finds:

$$k_1 = \sqrt{\frac{\sqrt{f(\omega)^2 + 4g(\omega)^2} + f(\omega)}{2}} \quad \text{and} \quad k_2 = \frac{g(\omega)}{k_1} \tag{25}$$

3.3 Attenuation, dispersion and velocities ratio

The attenuation and dispersion coefficients and velocities ratio are defined from k_1 and k_2 as indicated below:

$$k_1 = \omega / (c + u_0) \tag{26}$$

$$\alpha = 2\, k_2 \tag{27}$$

Numerically, these two values are obtained from the observed signal processing.

Determination of the attenuation. The method consists in determination of a decreasing exponential function in a e^{-k2x} form going through all the maxima A_i. k_2 is the attenuation of the signal, and it is given by:

$$k_2 = \frac{\ln \dfrac{A_i}{A_{i+1}}}{\lambda}$$

Determination of the dispersion. Dispersion is related to the jump of the pressure wave velocity when it propagates through the gas or the two-phase medium. The method consists of determination of the wavelength of the signal. Then, the wave celerity in the two-phase medium is deduced from:

$$c + u_0 = \lambda / Ts$$

The dimensionless attenuation and dispersion are defined by:

$$\tilde{\alpha} = \frac{2\,(c_0 + u_0)\, k_2}{C_m\, \omega} \tag{28}$$

$$\tilde{\beta} = \frac{\left(\dfrac{c_0 + u_0}{c + u_0}\right)^2 - 1}{C_m} \geq 0 \tag{29}$$

If the flow is at rest ($u_0 = 0$) one can notice that the expressions $\tilde{\alpha}$ and $\tilde{\beta}$ are the same ones given by Temkin and Dobbins[3]. In the other cases ($u_0 \neq 0$) this formula is suitable the propagation and the convection of the signal.

For a uniform concentration and a flow velocity given the pressure oscillations attenuation and dispersion due to the presence of particles depend on $\omega\, \tau_d$ only.

Determination of the velocities ratio. From relation (15), the modulus and the phase lag of the complex number $Z = u_p/u$ are found equal to:

$$|Z| = \frac{1}{\sqrt{(1 - k_2\, u_0\, \tau_d)^2 + (\omega - k_1\, u_0)^2\, \tau_d^2}} \tag{30}$$

$$\Phi = \mathrm{Arctg}\left(\frac{(\omega - k_1\, u_0)\, \tau_d}{1 - k_2\, u_0\, \tau_d}\right) \tag{31}$$

When the velocity of the flow is equal to zero, these two quantities become:

$$|Z| = \frac{1}{\sqrt{1 + \omega^2\, \tau_d^2}} \tag{32}$$

$$\Phi = \mathrm{Arctg}\,(\omega\, \tau_d) \tag{33}$$

The modulus of Z is, for a maximum selected, the ratio of the corresponding velocities. The phase lag is deduced, as soon as the distance Dl between two consecutive maxima has been measured, from the relation:

$$\Phi = \frac{2\,\pi\,\Delta l}{\lambda}$$

4. Numerical method

The numerical scheme is based on the Mac-Cormack explicit finite volume scheme. This scheme has the second order accuracy in time and space. It is well known[9-12], that this scheme in its finite difference form is appropriate for such computations.

For a given equation $U_t + F_x = H$, the two steps are:

Predictor:

$$\tilde{U}_j = U_j^n - \frac{\Delta t}{Vol} F_j^+ \vec{l}_j^{\,r} . \vec{i} + \Delta t\, H$$

Corrector:

$$U_j^{n+1} = \frac{1}{2}\left\{U_j^n + \tilde{U}_j - \frac{\Delta t}{Vol} F_j^- \vec{l}_j . \vec{i} + \Delta t\, H\right\}$$

where F^+ represents the flux term approximated by a forward difference, and F^- by a backward one, $\vec{l}_j$ being the positive or negative outward normal vector of cell j.

Owing to the explicit nature of this scheme, CFL stability criterion must be satisfied. The integration time step is chosen equal to 0.95 CFL. Numerical experiments have shown that the mesh needs 20 grid points per wavelength to an accurate solution.

5. Results

Attention has been focused on the case of an acoustic wave propagating through a one-dimensional, unsteady two-phase (gas-particles) flow in a duct. To simulate the acoustic signal, a periodic pressure oscillation is imposed at the inlet. The inlet pressure time dependence is given by $P = P_0 + \Delta P \cos(\omega t)$. The two-phase medium is at rest or have an equilibrium velocity $u_0 = u_{p0}$ in the duct. The duct is long enough to contain ten signal wavelengths, which is sufficient to assess the attenuation and the dispersion coefficients.

The common mean values retained for the next calculations are given in Table I.

Gas	Particles
$P_0 = 50$ bars	$\rho_{A1} = 1{,}766 kgm^{-3}$
$T_0 = 3{,}500$ K	Cpp = 0.68 Cp
$\gamma = 1.23$	Cm = 0.396
Cp = 2021.8 J kg^{-1} K^{-1}	T_{p0} 3,500 K
$\mu = 8.855\ 10^{-5}$kg m^{-1} s^{-1}	D_p = [2-40μm]
Pr = 0.8	
$c_0 = 1275.75$m s^{-1}	*Pressure signal*
R = 378.06 J kg^{-1} K^{-1}	$\Delta P/P_0 = 1$ or 0.1%
	ω = 1,000 Hz

Table I.
Mean values of two-phase medium and the pressure signal

The main interest of this topic is to show the influence of the flow velocity on the sound wave propagation. The theoretical results for the main pressure signal characteristics (attenuation, dispersion) are confronted with those obtained by the solution of the system of equations (6, 7). It was pointed out in the previous section that only the case without initial velocity ($u_0 = 0$ m/s) has an analytical solution. For a moving initial mixture, the solution for the signal is given by a fourth order complex polynomial equation. To obtain it, a numerical treatment is needed with some restrictions. Then, it is important to verify whether or not the finite volume method results are in good agreement with the analytical ones. Thus the finite volume method can be a good way to investigate the behaviour of an acoustic wave.

5.1 Two-phase medium at rest ($u_0 = 0$).

Some previous works[1] have already pointed out that particles motion are governed by gas motion, as $\omega \tau_d$ tends towards zero. On the other hand, as $\omega \tau_d$ exceeds over unit, inertial effects are no longer negligible. In the range $0 \leq \omega \tau_d \approx 1$ Stokes law can be used for the drag force. Moreover, to fall in the scope of the acoustics theory and to put back in the range where Stokes' assumptions are valid, the amplitude ratio of the pressure signal must be close to 0.1 per cent[13]. $\omega \tau_d$ appears to be the most significant characteristic of the two-phase medium.

Hereafter this parameter varies from 0 to 12. It corresponds to the 0-40μm range for the diameter. Owing to the fact that particles are small in comparison to the acoustic boundary layer thickness, Stokes law may be applied even for unsteady flows.

From Figure 1 (a, b, c, d), one can notice that there is a good agreement except for limiting values of $\omega \tau_d$ ($\omega\tau_d >> 1$ or $\omega\tau_d << 1$). That last point can be explained by a lack of accuracy in the determination of the wavelength value.

5.2 Case of a two-phase flow ($u_0 \neq 0$)

In this part numerical investigations of various two-phase flow are achieved. The mean velocity is equal to 0, 40 or 600m/s with a view to magnifying its influence.

One must remark that for low values of $\omega \tau_d$ (i.e. for very small particle diameters) the dynamic relaxation time is close to zero, therefore the particle fluctuations follow the gas fluctuations. In such a case, the drag force and the heat transfer do not disturb the pressure signal. As $\omega \tau_d$ increases inertia phenomena can be no longer neglected. That explains the bell shaped curves for attenuation (Figure 2a). The changes occurring in the neighbourhood of $\omega \tau_d = 1$ underscore the importance of particles in such phenomena as instabilities in engines[10].

Moreover, it is interesting to examine the values of the ratio of a pressure wave celerity and the mean particle velocity in the three cases shown in Table II.

In the first case ($u_0 = 0$m s^{-1}) the influence of wave celerity is much more important than particle velocity, consequently the attenuation of the signal is greater. In the case of a flow with a 600m s^{-1} velocity, wave celerity is of the same order of magnitude as particle velocity, and particles have a lower influence. In the limiting case where the wave celerity and the particle velocity would be the same, one expects no attenuation at all. As the flow velocity increases from 40 to 600m s^{-1}, attenuation is diminishing by 31 per cent (see Figure 2a).

Otherwise, as illustrated in Figure 2a, the greater the flow velocity, the greater the value of $\omega \tau_d$ for which a maximum attenuation occurs. As an example, for a flow velocity of 600m s^{-1}, the maximum attenuation is achieved for $\omega \tau_d$ equal to 1.7 (that means for particle of 18μm diameter), whereas $\omega \tau_d$ is equal to unit for a medium at rest (that means for particle of 12μm diameter).

From Figure 2b, it can be noticed that as $\omega \tau_d$ tends to infinity (high particle diameters), the dimensionless dispersion tends toward zero: particles no longer respond to gas velocity and temperature fluctuations, the sound celerity is no longer modified[14] and remains close to the gas phase value. This is confirmed by the relation (29). Conversely, when $\omega \tau_d$ tends to zero, as indicated previously, particle fluctuations follow the gas fluctuations and therefore the suspension behaves like a gas where sound celerity could be defined as[14]:

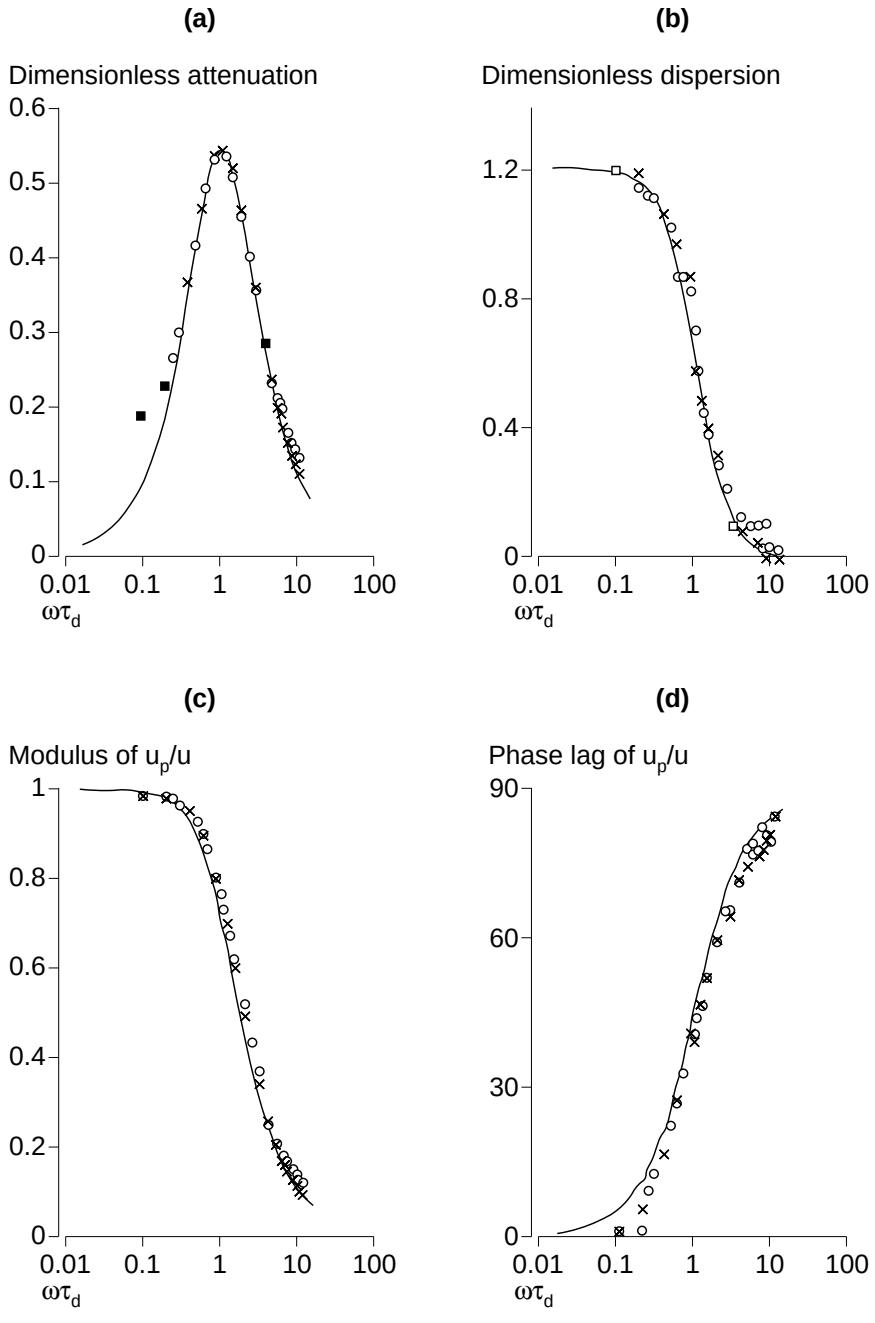

Figure 1. Evolution of (a) the dimensionless attenuation; (b) the dimensionless dispersion; (c) the modulus; (d) the phase lag of u_p/u versus $\omega\,\tau_d$ for a frequency of 1,000 Hz and for a medium at rest

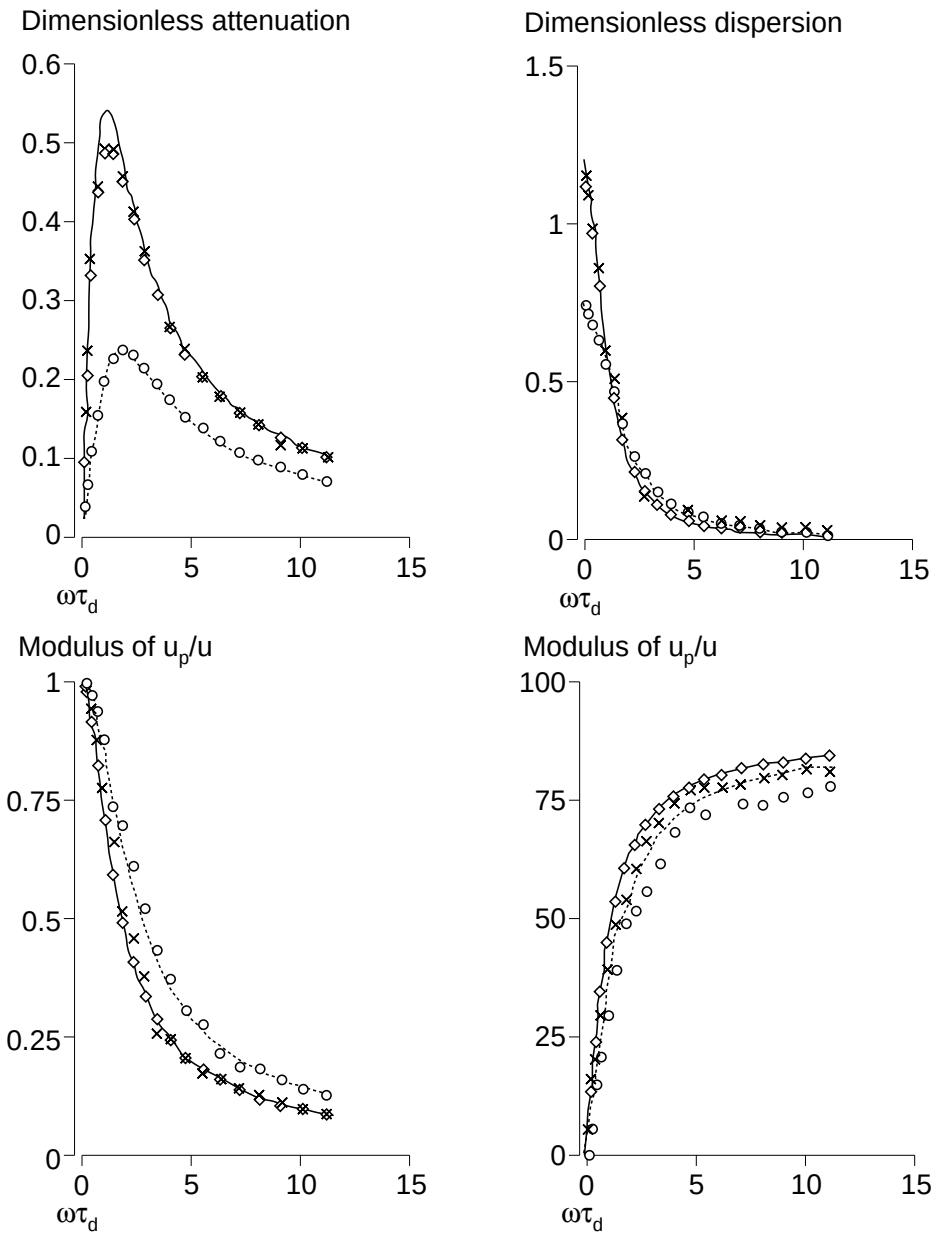

Figure 2.
Influence of the flow velocity on the evolution of (a) the dimensionless attenuation; (b) the dimensionless dispersion; (c) the modulus; (d) the phase lag of u_p/u versus $\omega\,\tau_d$

$$c_{0diph}^2 = \gamma_{diph} \hat{R} T_0 = c_0^2 \frac{1 + C_m \frac{C_{pp}}{C_p}}{(1 + C_m)\left(1 + \gamma C_m \frac{C_{pp}}{C_p}\right)} \quad (34)$$

From (29) and taking into account (34) it becomes:

$$\tilde{\beta} = \frac{\left(\frac{c_0 + u_0}{c_{0diph} + u_0}\right)^2 - 1}{C_m} \quad (35)$$

Table III shows the good agreement between analytical (35) and numerical (Figure 2a) dimensionless dispersion for an equilibrium suspension.

In Figure 2c (respectively 2d), it is shown that the higher the velocity is, the higher (respectively the lower) the modulus (respectively the phase lag) of the velocities ratio u_p/u is. In order to verify the adequacy of the behaviour of the Mac-Cormack numerical scheme when the medium is no longer at rest, numerical results from two-phase flow computations are confronted to the linearized theory results in the cases of both flow velocities 40m s^{-1} and 600m s^{-1} (see Figure 2).

Finally a very good agreement is achieved. This comparison confirms the high capability of the numerical procedure to simulate correctly an acoustic wave propagation through a two-phase flow.

Flow velocity	u_0 = 0m s^{-1}	u_0 = 40m s^{-1}	u_0 = 600m s^{-1}
$(u_0 + c_0)/u_{p0}$	Infinity	32.87	3.125

Table II. Flow velocity effect on the ratio $(u_0 + c_0)/u_{p0}$

Flow velocity (m s^{-1})	Theoretical value	Relation (35)
0	1.172	1.207
40	1.114	1.125
600	0.718	0.721

Table III. Theoretical and numerical dimensionless dispersion values for various flow velocities

6. Conclusions

Based on the study of pressure wave attenuation in a two-phase flow, the good capability of computational method to reproduce accurate results has been proved. From linearized theory one can obtain solutions for the pressure wave attenuation and dispersion, as well as for the modulus and the phase lag of the velocities ratio, versus frequency and particle diameter, assuming an uniform particles concentration. The good agreement between results derived from both

theory and numerical computations means that it is possible to expect valid numerical extensions to more complex configurations.

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